ANS coding replacing Huffman and AC – modern basic data representation

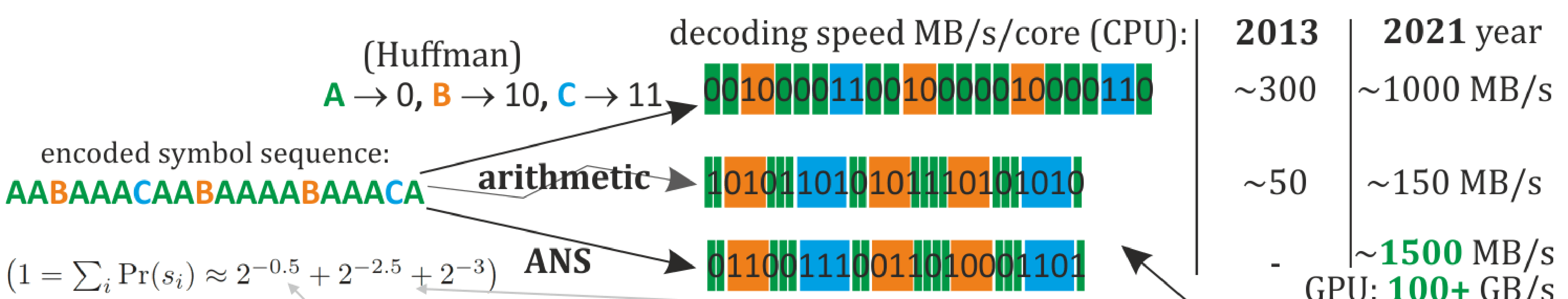
Asymmetric Numeral Systems **symbols ↔ final bits** of modern data compressors e.g.:

Apple LZFSE (**default in iPhones and Macs**), Facebook **ZSTD**: (e.g. in **Linux kernel**), CRAM 3.0 (**default DNA**), Google 3D Draco (e.g. Pixar), neural-network-based **JPEG XL** **~3x smaller than JPEG**, alpha, HDR, lossless – recently ISO standard, Chrome and maaaany others since 2007 – **large community just sharing work and ideas**

ANS solved difficulty: efficient processing of fractional bits

Jarek Duda

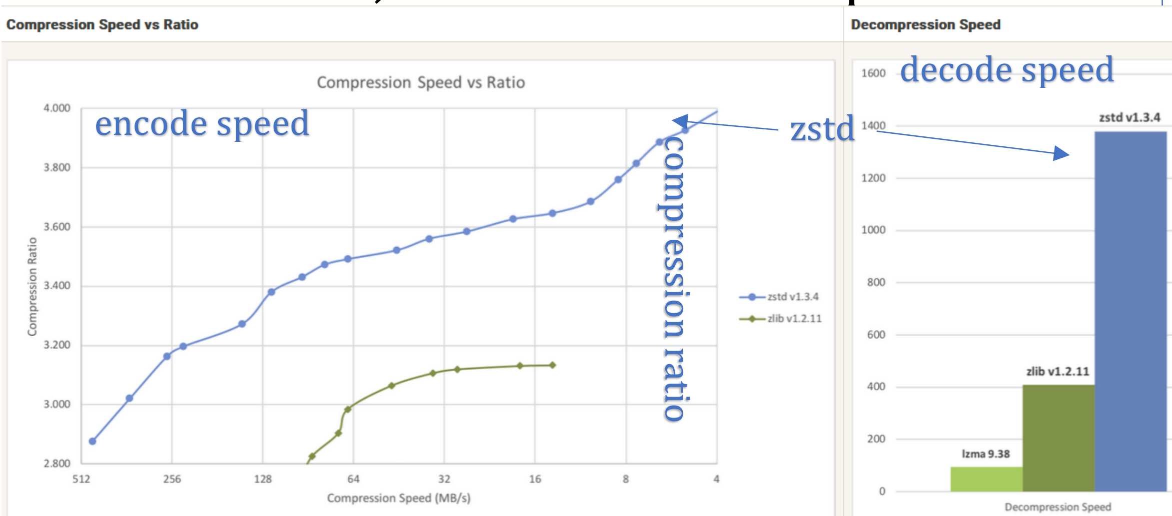
Data compression: data → **symbols** e.g. AABAAACAABAAAABAAACA → final bits:
frequent symbols: short representation, rare symbols: long ($\lg_2(1/\text{probability})$ bits)
ideally: 50% probability - 1 bit, 25% - 2 bits ... 'A': 70% prob. – ideally 0.5 **half of bit**:



Using half bit for 'A', at cost of increased length for 'B' and 'C', the encoding sequence is shorter

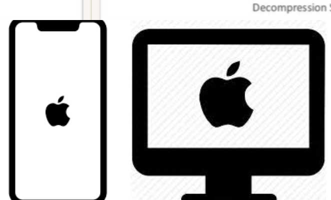
Facebook Zstandard – replaces **gzip**

e.g. in **Linux kernel**, lots of software:
3-5x faster, much better compression



Apple LZFSE –

default in iPhone, Mac



CRAM (in SAMtools) -
~default DNA compressor

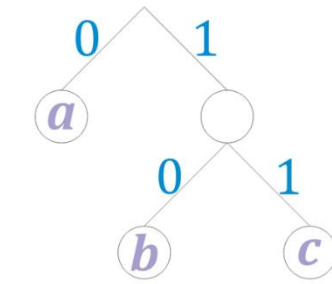
JPEG XL: to replace **JPEG** after 30 years
~3x smaller photos, images

Games: Oodle, **Microsoft** BCPack **DirectX**
+many more e.g. **Draco3D**, neural network

Saving time, transmission costs, energy, storage, hardware costs

10GB large **text** benchmark (2020, i9 9900K), 1GB wiki for 10 languages (ANS):

10GB -> **Size**	**encoding time**	**decoding time**	
5,034,758,325 bytes,	18.449 sec. -	**7**.311 sec.,	lz4 -1 (v1.9.2)
4,666,386,317 bytes,	26.686 sec. -	**4**.827 sec.,	lzturbo -10 -p0 (v1.2)
4,371,496,854 bytes,	46.907 sec. -	**7**.282 sec.,	lz4x -1 (v1.60)
3,909,521,247 bytes,	32.603 sec. -	**11**.287 sec.,	lizard -40 (v1.0.0)
3,823,273,187 bytes,	136.146 sec. -	**59**.070 sec.,	**gzip** -1 (v1.3.12)
3,770,151,519 bytes,	34.216 sec. -	**26**.236 sec.,	brotli -q 0 (v1.0.7)
3,642,089,943 bytes,	28.752 sec. -	**10**.717 sec.,	**zstd** -1 (v1.4.5) LZ + **tANS**/huf
3,660,882,443 bytes,	767.399 sec. -	**7**.633 sec.,	lz4x -9 (v1.60)
3,237,812,198 bytes,	392.835 sec. -	**53**.771 sec.,	**gzip** -9 (v1.3.12)
3,095,248,795 bytes,	137.881 sec. -	**20**.738 sec.,	brotli -q 4 (v1.0.7)
3,078,914,611 bytes,	240.124 sec. -	**9**.381 sec.,	**zhuff** -c2 -t1 (v0.99beta), LZ4 + **tANS**
3,065,081,662 bytes,	50.724 sec. -	**12**.904 sec.,	**zstd** -4 --ultra --single-thread (v1.4.5)
2,660,370,879 bytes,	153.103 sec. -	**19**.993 sec.,	**lzturbo** -32 -p0 (v1.2), LZ + **tANS**
2,639,230,515 bytes,	561.791 sec. -	**11**.774 sec.,	**zstd** -12 --ultra --single-thread(v1.4.5)
2,357,818,671 bytes,	3,953.092 sec. -	**34**.300 sec.,	rar -m5 -ma5 -mt1 (v5.80)
2,337,506,087 bytes,	2,411.038 sec. -	**11**.971 sec.,	**zstd** -18 --ultra --single-thread(v1.4.5)
2,220,027,943 bytes,	7,439.064 sec. -	**22**.690 sec.,	brotli -q 10 (v1.0.7)
2,080,479,075 bytes,	4,568.550 sec. -	**12**.934 sec.,	**zstd -22** --ultra --single-thread(v1.4.5)
2,059,053,547 bytes,	4,909.124 sec. -	**55**.188 sec.,	7z -t7z -mx9 -mmt1 (v19.02) - LZMA
1,973,568,508 bytes,	6,626.946 sec. -	**89**.762 sec.,	arc -m9 -mt1 (v0.67)
1,921,561,064 bytes,	17,200.759 sec. -	**27**.147 sec.,	brotli -q 11 --large_window=30 (v1.0.7)
1,899,403,918 bytes,	1,327.809 sec. -	**375**.295 sec.,	nz -cO -t1 (v0.09 alpha)
1,722,407,658 bytes,	778.796 sec. -	**401**.317 sec.,	m99 -b1000000000 -t1 (beta)
1,675,874,699 bytes,	781.839 sec. -	**198**.309 sec.,	bwtturbo -59 -t0 (v20.2)
1,644,097,084 bytes,	21,097.196 sec. -	**93**.130 sec.,	**razor** (v1.03.7) - **adaptive 4bit rANS**
1,638,441,156 bytes,	1,030.489 sec. -	**640**.502 sec.,	bsc -m0 -b1024 -e2 -T (v3.1.0)
1,632,628,624 bytes,	1,146.133 sec. -	**1,284**.451 sec.,	bcm -9 (v1.40)
1,450,364,034 bytes,	2,701.335 sec. -	**2,433**.988 sec.,	mcm -x -m11 (v0.83)

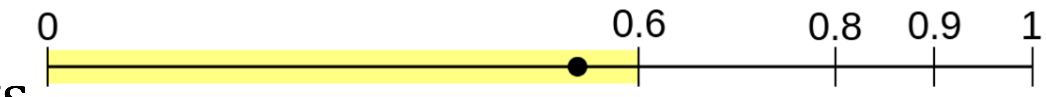


Brief history: prefix codes leading to Huffman coding (1952)

Shannon-Fano coding (Fano, 1949)

Shannon-Fano-Elias coding (Elias, 1963)

produce bits for each symbol 2.3 →[] 3 bits



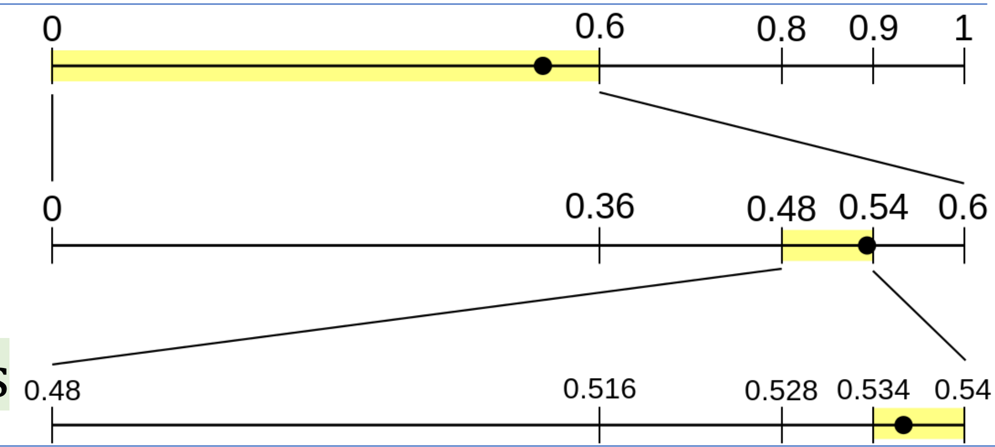
Arithmetic coding (**AC**) (history)

Jorma Rissanen, Richard Pasco (1976)

Nigel Martin (1979) many independently

lots of patents ... widely used in h.264 (**2004**)

produce bits after accumulating many symbols



ANS: simpler, cheaper alternative: single state $x \in N$

x'	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	…
x, $s=0$	0				1				2				3				4			
x, $s=1$		0	1	2		3	4	5		6	7	8		9	10	11		12	13	

2006 – first ANS variant in my physics MSc thesis (translation + later tANS)

2007,8 – tANS variant, implementations by Matt Mahoney, Andrew Polar

2013: Yann Collet tANS/FSE/zhuff, my often cited paper later introducing rANS

2014: Fabian Giesen rANS, James Bonfield very fast **Markov rANS** + CRAM

2015: Zstandard later Facebook, **Adaptive rANS**, Apple LZFSE and many more

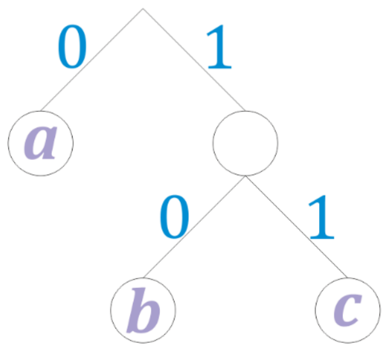
Will ANS remain nearly default in **future?** Has weakness: LIFO → checksum

symbol sequence
complex probabilities

abaaabaabaa ⟷ 0100101

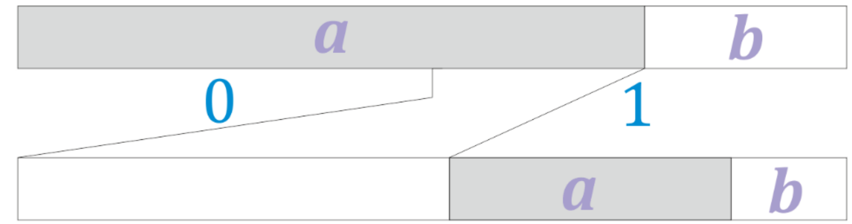
bit sequence
Pr(0)=Pr(1)=1/2

Past: compromise



(prefix,) **Huffman coding**
(also unary, Golomb, Elias, etc.)
fast (>300MB/s/core)
no multiplication, needs sorting
but **inaccurate:** Pr(*s*) ~ 2^{-r}
e.g. for Pr(*a*)=0.01, Pr(*b*)=0.99
uses **1** bit/symbol

or ?



arithemtic/range coding
slow (<< 100MB/s/core)
uses multiplication
uses nearly **accurate** Pr(*s*)
e.g. for Pr(*a*)=0.01, Pr(*b*)=0.99
uses ~**0.08** bits/symbol

Now: ANS

tANS: **tabled** - no multiplication
"Huffman generalized to fractional bits"
also allows for simultaneus encryption

rANS: **range** - direct replacement
of arithmetic/range coding: with
smaller state, less multiplications

mainly used for
smaller models,
fixed
distributions

fast (> 500MB/s/core)
uses nearly **accurate** Pr(*s*)
e.g. for Pr(*a*)=0.01, Pr(*b*)=0.99
uses ~**0.08** bits/symbol

mainly used for
larger models,
adaptive
distributions

***x* state stores information**

binary system: to encode symbol s:

rule: ‚new state' = 2• ‚old state' + s

new state x / old state	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	...
$s = 0$	0		1		2		3		4		5		6		7		8		9	
$s = 1$		0		1		2		3		4		5		6		7		8		

example state: 1 $\xrightarrow{s=0}$ 2 $\xrightarrow{1}$ 5 $\xrightarrow{1}$ 11 $\xrightarrow{1}$ 23 $\xrightarrow{1}$ 47

asymmetric binary system (ANS):

rule: ‚new state' = number ‚old state' appearance of s

new state x / old state	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	...
$s = 0$	0				1				2				3				4			
$s = 1$		0	1	2		3	4	5		6	7	8		9	10	11		12	13	

example stan: 1 $\xrightarrow{s=0}$ 4 $\xrightarrow{1}$ 6 $\xrightarrow{1}$ 9 $\xrightarrow{1}$ 13 $\xrightarrow{1}$ 18

symbol sequence „01111", encoded by binary system as 47, ANS as shoorter (cheaper to write) 18 thanks to better agreement with: 1 more frequent than 0

ANS: $\boldsymbol{x \to \approx x/\Pr(s)}$ while encoding symbol s

Redefine even/odd subsets according to densities

$x \to x$-th appearance of 'even' ($s = 0$) or 'odd' ($s = 1$)

rANS variants: repeating division in ranges, e.g. of size 4:

$\bar{s}(x) = 0$ if $\mathrm{mod}(x,4) = 0$, else $\bar{s}(x) = 1$

to decode or encode 1, localize quadruple ($\lfloor x/4 \rfloor$ or $\lfloor x/3 \rfloor$)

if $\bar{s}(x) = 0$, $D(x) = (0, \lfloor x/4 \rfloor)$ else $D(x) = (1, 3\lfloor x/4 \rfloor + \mathrm{mod}(x,4) - 1)$

$C(0,x) = 4x$ $\quad$ $C(1,x) = 4\lfloor x/3 \rfloor + 1 + \mathrm{mod}(x,3)$

$\boldsymbol{x' \approx x/\Pr(s)}$

x'		0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18 …
x	$s=0$	0				1				2				3				4		
	$s=1$		0	1	2		3	4	5		6	7	8		9	10	11		12	13

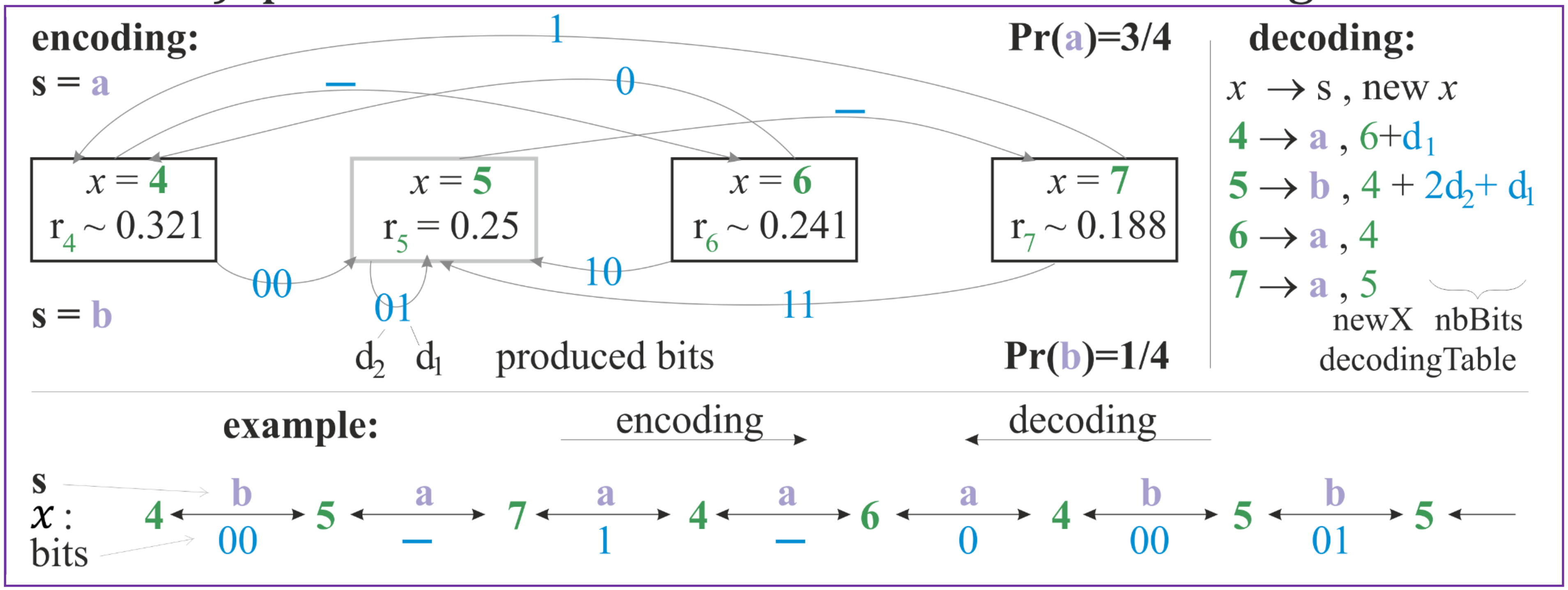
e.g. $x = 1 \xrightarrow{s=0} 4 \xrightarrow{1} 6 \xrightarrow{1} 9 \xrightarrow{1} 13 \xrightarrow{1} 18$

+ renormalization – make $x \in I$ e.g. $I=\{4,5,6,7\}$ **below**, $I = [2^{16}, 2^{32}-1]$ **rANS**

tANS (**tabled**, 2007): put into table with renormalization, building **automaton**

example:
{a,b}
symbols
$x \in \{4, 5, 6, 7\}$
states

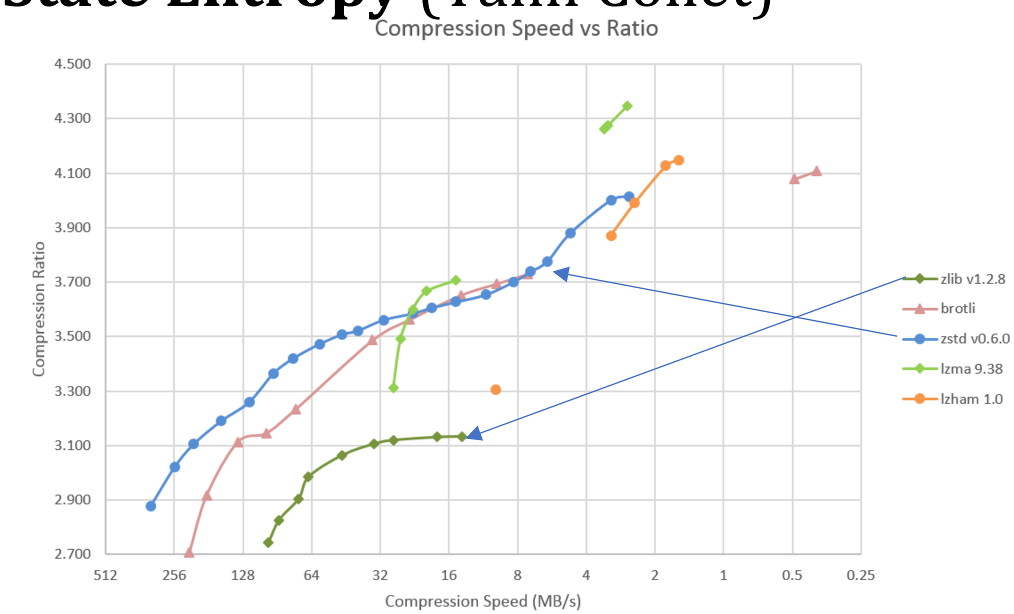
Pr(a) > 1/2
a **carries**
< 1 bit

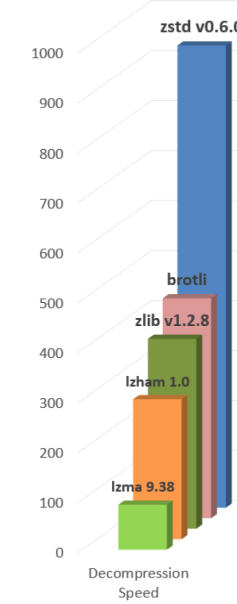


tANS used e.g. as [FSE](#) – Finite State Entropy (Yann Collet)

(gzip→) **in [Zstd](#)** – widely used
e.g. Facebook, **Linux kernel**,
[lots of software](#), corporations

[Apple LZFSE](#) –
default in iPhone, Mac





tANS (2007) - **fully tabled behavior** for given probability distribution

Apple LZFSE, **Facebook ZSTD**, **lzturbo** … "**Huffman + fractional bits**"

fast: no multiplication (**FPGA**!), less memory efficient (~8kB for 2048 states)

static in ~32kB blocks, costly to update (rather needs rebuilding),

allows for simultaneous encryption (CSPRNG to perturb symbol spread)

tANS decoding step	**Encoding step** (for symbol s)
t = decodingTable[x];	nbBits = (x + nb[s]) >> r ;
writeSymbol(t.symbol);	writeBits(x, nbBits);
x = t.newX + readBits(t.nbBits);	x = encodingTable[start[s] + (x >> nbBits)];

rANS (2013) – needs one multiplication per symbol, good for SIMD/GPU

CRAM (DNA), **RAZOR**, **BB-ANS**(neural networks), **JPEG XL**, **GPU** (100+ GB/s)

Works directly on probabilities – more flexible, adaptivity

more memory effective – especially for large alphabet and precision, Markov

rANS decoding step ($mask = 2^n - 1$)	**Encoding step** (s) ($msk = 2^{16} - 1$, $d = 32\text{-}n$)
s = **symbol**(x & $mask$); writeSymbol(s);	if($x \geq (f[s] << d)$)
$x = f[s]\ (x >> n) + (x\ \&\ mask) - c[s]$;	{write16bits(x & msk); x >>= 16; }
if($x < 2^{16}$) $x = x << 16$ + read16bits();	$x = \lfloor x / f[s] \rfloor << n + (x \% f[s]) + c[s]$;

MB/s: **tANS/FSE: 380/500** **rANS**: **500/1500** … **GPU** rANS: **100+ GB/s**